

# $N^*$ Masses from QCD Sum Rules

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(Received: October 29, 2001)

We report  $N^*$  masses in the spin-1/2 and spin-3/2 sectors using the method of QCD Sum Rules. They are based on three independent sets derived from generalized interpolating fields. The predictive ability of each sum rule is examined by a Monte-Carlo based analysis procedure in which all three phenomenological parameters (mass, coupling, threshold) are extracted simultaneously. A parity projection technique is also studied.

The QCD Sum Rule method [1] is a time-honored method that has proven useful in revealing a connection between hadron phenomenology and the non-perturbative nature of the QCD vacuum via only a few parameters (the vacuum condensates). It has been successfully applied to a variety of observables in hadron phenomenology, providing valuable insights from a unique, QCD-based perspective, and continues an active field (try a keyword search with 'QCD Sum Rule'). The method is analytical (no path integrals!), is physically transparent (one can trace back term by term which operators are responsible for what), and has minimal model dependence (Borel transform, and a continuum threshold). The accuracy of the approach is limited due to limitations inherent in the operator-product-expansion (OPE), but well understood.

Our goal is to explore the possibility of using the method to understand the  $N^*$  spectrum. The calculation of baryon masses in the approach is not new [2, 3, 4, 5, 6]. Here we focus on the excited states and emphasize the predictive ability of the method for  $N^*$  properties based on careful analysis, using a rigorous Monte Carlo-based [7] numerical analysis procedure that treats all three phenomenological parameters (mass, coupling, threshold) as free parameters and extracts them simultaneously with error bars. In particular, we study the low-lying states in the spin-1/2 and spin-3/2 sectors with both positive and negative parity. A similar analysis for the baryon decuplet has been done [8].

The starting point is the time-ordered, two-point correlation function in the QCD vacuum:

$$\Pi(p) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ \eta(x) \bar{\eta}(0) \} | 0 \rangle, \quad (1)$$

where  $\eta$  is the interpolating field that has the quantum numbers of the baryon under consideration. We consider the most general current for the nucleon with  $I(J^P) = \frac{1}{2} \left( \frac{1}{2}^+ \right)$ ,

$$\eta_{1/2}^N(x) = -2 \left[ \epsilon_{abc} \left( u^{aT}(x) C \gamma_5 d^b(x) \right) u^c(x) + \beta \epsilon_{abc} \left( u^{aT}(x) C d^b(x) \right) \gamma_5 u^c(x) \right]. \quad (2)$$

Here  $C$  is the charge conjugation operator, the superscript  $T$  means transpose, and  $\epsilon_{abc}$  makes it color-singlet. The real parameter  $\beta$  can be varied to achieve maximal overlap with the state in question. The choice advocated by Ioffe [2] and often used in QCD sum rules studies corresponds to  $\beta = -1.0$ . It is well-known that a baryon interpolating field couples to states of both parities, despite having an explicit parity by construction. The results below will show that  $\beta$  can be varied to saturate a sum rule with either positive or negative parity states. For states with  $I(J^P) = \frac{1}{2} \left( \frac{3}{2}^+ \right)$ , we consider

$$\eta_{3/2,\mu}^N(x) = \epsilon_{abc} \left[ \left( u^{aT}(x) C \sigma_{\rho\lambda} d^b(x) \right) \sigma^{\rho\lambda} \gamma_\mu u^c(x) - \left( u^{aT}(x) C \sigma_{\rho\lambda} u^b(x) \right) \sigma^{\rho\lambda} \gamma_\mu d^c(x) \right]. \quad (3)$$

The interpolating fields for  $\Sigma$ ,  $\Lambda$  and  $\Xi$  can be obtained by appropriate substitutions of quark fields under  $SU(3)$  color symmetry or flavor symmetry.

With two kinds of interpolating fields, three possible correlation functions can be constructed: the correlator of generalized spin-1/2 currents  $\eta_{1/2}$  and  $\bar{\eta}_{1/2}$ , the mixed correlator of generalized spin-1/2 current  $\eta_{1/2,\mu} = \gamma_\mu \gamma_5 \eta_{1/2}$  and the spin-3/2 current  $\bar{\eta}_{3/2,\nu}$ , and the correlator of spin-3/2 currents  $\eta_{3/2,\mu}$  and  $\bar{\eta}_{3/2,\nu}$ . From them, 11 independent sum rules emerge which can be used to study  $1/2\pm$  and  $3/2\pm$  states.

Table 1 shows the predictions for  $1/2+$  states from the chiral-odd sum rules at the tensor structure  $\gamma_\mu p_\nu \hat{p}$ , using the Monte-Carlo analysis. Sum rules fall into two categories: one with odd-dimension operators (chiral-odd) and the other with even-dimension operators (chiral-even). The predictions compare favorably with the observed values, with an accuracy of about 100 MeV. The couplings come as by-products which are useful in the calculation of matrix elements because they enter as normalization. Table 2 shows the predictions for  $3/2-$  states.

One drawback in the conventional approach is that states with both parities contribute in the sum rules. Although sometimes one can saturate a sum rule with a certain parity by adjusting  $\beta$ , as done above, it is desirable to separate the two parities exactly. This can be achieved by replacing the time-ordering operator  $T$  in the correlation function in Eq. (1) with  $x_0 > 0$ , and constructing sum rules in the complex  $p_0$ -space in the rest frame ( $\vec{p} = 0$ ) [9]. This is equivalent to a parity projection technique used in lattice QCD calculation of  $N^*$  masses [10]. Table 3 shows the predictions for  $1/2-$  states in this method. The results are much improved, as indicated by the smaller error bars and very wide Borel regions. The agreement with experiment is excellent. To further investigate the origin of splittings between parity partners, we show in Figure 1 the mass splittings between  $N_{\frac{1}{2}-}$  and  $N_{\frac{1}{2}+}$  as a function of the quark condensate (the order parameter of spontaneous chiral symmetry breaking). One can see a clear decrease in the splitting with decreasing quark condensate, in the range that the sum rule does not break down.

In conclusion, we demonstrated the predictive power of QCD sum rules for  $N^*$  masses in the low-lying  $1/2\pm$  and  $3/2-$  sectors, with an accuracy on the order of 5 to 10%. The parity separation method is promising. We are extending it to the spin-3/2 sector. More analysis is under way to understand the details of the splitting patterns across particle channels and parities, in terms of explicit and dynamical chiral symmetry breaking.

**Acknowledgments:** This work is supported in part by U.S. Department of Energy under Grant DE-FG03-93DR-40774.

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Table 1: Predictions for  $1/2+$  states from the chiral-odd sum rules at the structure  $\gamma_\mu p_\nu \hat{p}$ .

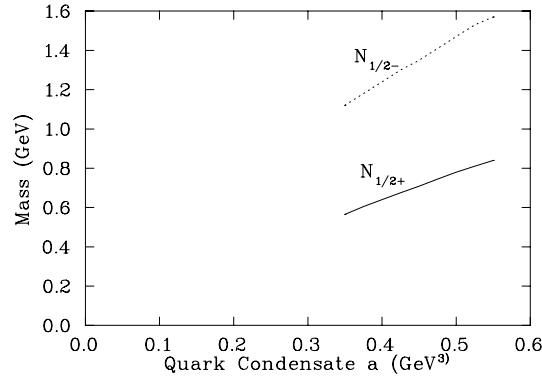
Sum Rule	Region (GeV)	$w$ (GeV)	$\tilde{\lambda}_{1/2}\tilde{\alpha}_{1/2}$ (GeV <sup>6</sup> )	Mass (GeV)	Exp. (GeV)
$N_{\frac{1}{2}+}(\beta = +1.0)$	1.06 to 1.46	$1.31 \pm .22$	$1.13 \pm .53$	$1.06 \pm .11$	0.938
$\Sigma_{\frac{1}{2}+}(\beta = +1.0)$	1.12 to 1.53	$1.48 \pm .23$	$1.62 \pm 0.68$	$1.16 \pm .12$	1.193
$\Xi_{\frac{1}{2}+}(\beta = +1.0)$	1.35 to 1.80	$1.69 \pm .27$	$2.61 \pm 1.20$	$1.32 \pm .14$	1.318
$\Lambda_{\frac{1}{2}+}(\beta = +1.0)$	1.28 to 1.72	$1.53 \pm .24$	$0.66 \pm 0.28$	$1.23 \pm .12$	1.116

 Table 2: Predictions for  $3/2-$  states from the chiral-odd sum rules at the structure  $g_{\mu\nu}$ .

Sum Rule	Region (GeV)	$w$ (GeV)	$\tilde{\lambda}_{3/2}^2$ (GeV <sup>6</sup> )	Mass (GeV)	Exp. (GeV)
$N_{\frac{3}{2}-}$	0.95 to 1.17	$1.65 \pm .24$	$27.6 \pm 11.8$	$1.44 \pm .13$	1.520
$\Sigma_{\frac{3}{2}-}$	1.29 to 1.36	$1.91 \pm .25$	$46.6 \pm 20.1$	$1.69 \pm .14$	1.580
$\Xi_{\frac{3}{2}-}$	1.30 to 1.39	$2.19 \pm .27$	$84.8 \pm 42.9$	$1.84 \pm .16$	1.820
$\Lambda_{\frac{3}{2}-}$	1.22 to 1.32	$2.01 \pm .25$	$19.8 \pm 9.3$	$1.71 \pm .15$	1.690

 Table 3: Predictions for  $1/2-$  states from the new method where parity is exactly separated.

Sum Rule	Region (GeV)	$w$ (GeV)	$\tilde{\lambda}_{3/2}^2$ (GeV <sup>6</sup> )	Mass (GeV)	Exp. (GeV)
$N_{\frac{1}{2}-}(\beta = +1.1)$	0.80 to 1.80	$2.26 \pm .08$	$4.58 \pm .38$	$1.53 \pm .05$	1.535
$\Sigma_{\frac{1}{2}-}(\beta = +1.1)$	0.80 to 1.90	$2.35 \pm .06$	$5.74 \pm .69$	$1.63 \pm .07$	1.620
$\Xi_{\frac{1}{2}-}(\beta = +1.1)$	0.80 to 1.80	$2.38 \pm .06$	$5.42 \pm .68$	$1.61 \pm .08$	1.620
$\Lambda_{\frac{1}{2}-}(\beta = +1.1)$	0.90 to 1.80	$2.49 \pm .04$	$7.06 \pm .55$	$1.67 \pm .05$	1.670


 Figure 1: Mass splitting between  $N_{\frac{1}{2}-}^*$  and  $N_{\frac{1}{2}+}$  as a function of the quark condensate parameter  $a = -(2\pi)^2 \langle \bar{q}q \rangle$ . The physical point corresponds to  $a = 0.52 \text{ GeV}^3$ .